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WIENER INDEX OF TREES WITH GIVEN NUMBER OF VERTICES OF ODD DEGREE SEQUENCE

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ABSTRACT

The wiener index of a graph $W(G)$ is the sum of the distance between all pairs of vertices $d_G(u,v)$ where $d_G(u,v)$is the number of edges in the shortest path connecting vertices u and v. We characterize trees with wiener index of the given number of vertices with odd degrees.

INTRODUCTION - BASIC DEFINITIONS AND REMARKS

Definition-1

The degree $deg_G(v)$ of a vertex $v \in V(G)$ is the number of edges incident to v.

Definition-2

The degree sequence of G is a vector $\left(deg_G(v_1), deg_G(v_2), \dots, deg_G(v_n)\right) with\ deg_G(v_1) \geq deg_G(v_2) \geq \cdots \geq deg_G(v_n)$ and $n = |V(G)|$.

Definition-3

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The distance $d_G(u,v)$ between two vertices $u, v \in V(G)$ is the minimum number of edges on a path in G between u and v.

Definition-4

Let G be a simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. Let S_n and P_n denote the star and path with n vertices. The distance of a vertex v, denoted by $d_G(v)$ which is the sum of distances between v and all other vertices of G. The distance between vertices u and v of G is denoted by $d_G(u,v)$. The Wiener index of a connected graph G is defined as

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u,v)$$

and the average distance $\mu(G)$ between the vertices of G by

$$\mu(G) = \frac{W(G)}{\binom{|V(G)|}{2}}, \qquad \text{where } \binom{|V(G)|}{2} = \frac{V(G)!}{2!(V(G)-2)}$$

Example-1:

Let G be the graph shown in Figure (a).

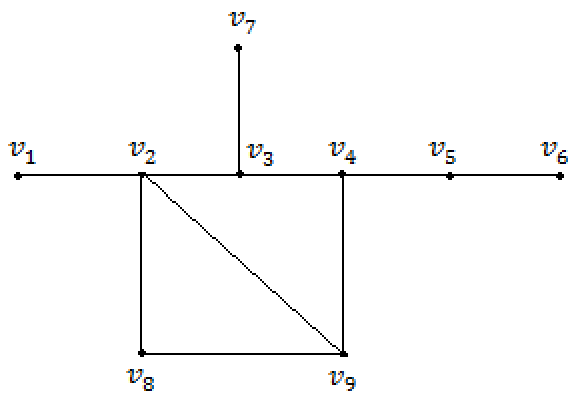


Figure (a): Connected graph G.

Then the wiener index of graph G is

$$W(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u,v)$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d_G(v_i, v_j)$$

$$= \sum_{i=1}^{8} \sum_{j=i+2}^{9} d_G(v_i, v_j)$$

$$\begin{aligned} = & d_G(v_1,v_2) + d_G(v_1,v_3) + d_G(v_1,v_4) + d_G(v_1,v_5) + d_G(v_1,v_6) + d_G(v_1,v_7) \\ & + d_G(v_1,v_8) + d_G(v_1,v_9) + d_G(v_2,v_3) + d_G(v_2,v_4) + d_G(v_2,v_5) + d_G(v_2,v_6) \\ & + d_G(v_2,v_7) + d_G(v_2,v_8) + d_G(v_2,v_9) + d_G(v_3,v_4) + d_G(v_3,v_5) + d_G(v_3,v_6) \\ & + d_G(v_3,v_7) + d_G(v_3,v_8) + d_G(v_3,v_9) + d_G(v_4,v_5) + d_G(v_4,v_6) + d_G(v_4,v_7) \\ & + d_G(v_4,v_8) + d_G(v_4,v_9) + d_G(v_5,v_6) + d_G(v_5,v_7) + d_G(v_5,v_8) + d_G(v_5,v_9) \\ & + d_G(v_6,v_7) + d_G(v_6,v_8) + d_G(v_6,v_9) + d_G(v_7,v_8) + d_G(v_7,v_8) \\ & + d_G(v_8,v_9) \end{aligned}$$

$$= 87$$

Note:

Every nontrivial tree has at least two vertices of degree one.

Theorem-1:

Every tree with only odd degree vertices have even number of vertices.

Proof:

We know that for any graph G

$$\sum_{v \epsilon V(G)} deg_G(v) = 2|E(G)|$$

This implies that each tree which has only vertices of odd degree must have even number of vertices. We can see that in the following figure (b).

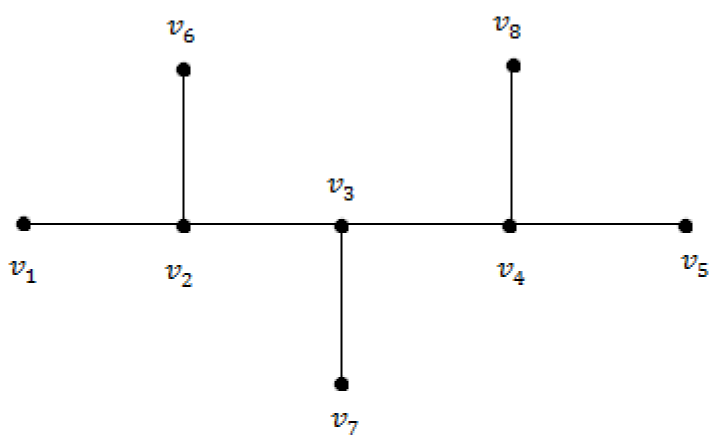


Figure (b): The tree with odd vertex degrees having even number of vertices.

Theorem-2:

Every tree with vertex of odd degree has even wiener index.

Proof:

We can prove this theorem using theorem-1.

Let us consider a wiener index of tree with all the n-vertices having odd number of degrees. We know that

$$W(G) = \sum_{\{u,v\} \subseteq V(G)} d_G(u, v)$$

(From figure (b) the wiener index W(G) = 64)

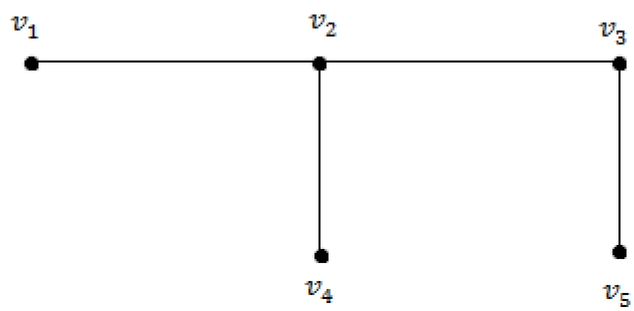
= Even

Corollary:

Converse part of the above theorem is not necessarily true.

Example:

Consider a tree with even wiener index.



Figure(c): Graph with even Wiener index=18

Here the degree of v_3 is even (2).

Conclusion:

We conclude that a tree of Wiener index given number of vertices of odd degree has even number of vertices. Finally we characterized the wiener index with odd degree sequence.

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